A 3-Dimensional Broadcast Protocol with Optimized Transmission Efficiency in Industrial Wireless Networks

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Abstract

Wireless technologies are promising for industrial automation and distributed control due to the properties of mobility, reduced deployment and maintenance costs, and so on. By the technologies, wireless networks are formed for integrating processes and devices of industrial systems. Broadcasting is one of the most important operations in the wireless network for disseminating information throughout the entire network. Flooding is a simple mechanism to realize broadcasting, but it has high redundancy of retransmissions, leading to low transmission efficiency. Many broadcast protocols have been proposed for pursuing optimized transmission efficiency for wireless networks hypothetically deployed on the 2-dimensional (2D) plane. In some industrial applications (e.g., a multi-storey factory), however, wireless networks are deployed in the 3D space. In this paper, we derive the upper bound of 3D transmission efficiency and propose a 3D broadcast protocol, called 3DOBP (3-Dimensional Optimized Broadcast Protocol), to achieve optimized transmission efficiency by partitioning the 3D space into multi-layer hexagonal prisms of a hexagon ring pattern in each layer. As we will show, the transmission efficiency of the proposed protocol is \(1/\pi\), which is better than those of polyhedron-filling 3D broadcasting approaches using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons. To the best our knowledge, the proposed broadcast protocol is the one with the highest 3D transmission efficiency so far.

Keywords: broadcasting, transmission efficiency, wireless networks, industrial applications

1. Introduction

Wireless technologies are promising for industrial automation and distributed control due to the properties of mobility, reduced deployment and maintenance costs, and so on [18]. Potential industrial applications using wireless technologies are coordination of mobile robots or autonomous vehicles, and industrial wireless sensor networks [6]. Some protocols are proposed to help form wireless networks for integrating processes and devices of industrial applications. For example, International Electrotechnical Commission (IEC) has approved WirelessHART [19], an extension of HART (Highway Addressable Remote Transducer) protocol for intelligent process measurement and
control instrumentation, as a Publicly Available Specification (IEC/PAS 62591Ed.1) to form wireless networks for industrial systems. It is now feasible to build a wireless network as the communication and distributed control basis of modern industrial systems.

Broadcasting is one of the most important operations for disseminating data or control signals among nodes of a wireless network. Flooding is an intuitive approach to realize broadcasting, in which each node retransmits a packet when receiving it for the first time. Flooding is simple and is highly reliable; however, it may cause the broadcast storm problem [12] and has low transmission efficiency due to redundancy of retransmissions. As shown in [10], the theoretical upper bound of transmission efficiency is 0.61 for wireless networks deployed on the 2-dimensional (2D) plane.

Some geometry-based broadcast protocols [5, 10, 11, 15] for 2D wireless networks have been proposed to pursue optimized transmission efficiency. Among them, the Optimized Broadcast Protocol (OBP) proposed in [11] achieves the highest transmission efficiency 0.55, which is about 90% of the theoretical upper bound. In OBP, a node relies on hexagon rings centered at the broadcasting source node to decide if it should retransmit a broadcast packet when it receives the packet for the first time.

In some industrial system environments, the wireless network is deployed in the 3-dimensional (3D) space instead of the 2D plane. A wireless network consisting of devices distributed in different floors of a multi-storey factory is a typical example. In such a 3D wireless network, the relationship between wireless communicating nodes should be 3D rather than 2D. This motivates us to investigate broadcasting and the transmission efficiency for 3D wireless networks.

Some papers [1-3, 7, 16] study the arrangement of nodes to fully cover a given space with optimized number of nodes. Most of them [1-3, 7] are based on the regular polyhedron filling approach, which partitions the space into 3D cells, each being a polyhedron located by a node in the center. The paper [1] investigated how to fill the space with regular polyhedrons, such as cubes, hexagonal prisms, rhombic dodecahedrons, and truncated octahedrons. The cube is used as a fundamental element to model a 3D wireless network in [7]. The hexagonal prism, rhombic dodecahedron are studied in [2] and [3] for planning base stations to fully cover a 3D wireless network. The paper [16] proposes using the body-center cubic (BCC) structure to fill the space; interestingly, filling space by BCCs is equivalent to filling space by truncated octahedrons.

When the center nodes of neighboring cells can communicate with each other, the polyhedron filling approach can be transformed into a 3D broadcast protocol by demanding the center node in
each cell to retransmitting the broadcast packet. We can thus design 3D broadcast protocols with the help of the above-mentioned regular polyhedrons to achieve good transmission efficiency. Such protocols have 100% coverage ratio, the ratio of the network space that is covered by the transmission ranges of the source node or the retransmitting nodes, when the node density is sufficiently high. The paper [4] adopts a different strategy to designing a 3D broadcast protocol, called Air to Air Communication Protocol (AACP), which requires vertex nodes, instead of center nodes, in hexagonal prisms to retransmit the broadcast packet for better transmission efficiency. Note that a similar protocol is also proposed in [14]. As we will show the AACP protocol does not achieve 100% coverage ratio even when the node density is very high.

![Fig. 1 Illustration of the upper bound case of transmission efficiency in the 3D space](image)

In this paper, we generalize the definition of transmission efficiency for 3D wireless networks. To take two communicating nodes A and B in Fig. 1 as an example, the transmission efficiency is the ratio of the effective communication area (the region covered by spheres \( S_A \) or \( S_B \), i.e., \(|S_A \cup S_B|\)) over the total communication region (the summation of volumes of \( S_A \) and \( S_B \), i.e., \(|S_A|+|S_B|\)), where \( S_A \) and \( S_B \) are the spheres centered respectively at A and B with the radius of R, the transmission range. When the distance between nodes A and B equals to the transmission range R, transmission efficiency reaches the upper bound. As we will show, the theoretical upper bound is 0.84 in the 3D space.

We further propose an optimized broadcast protocol, called 3D optimized broadcast protocol (3DOBP), by partitioning the 3D space into multi-layer hexagonal prisms. As shown in Fig. 2(a), the network space is partitioned into multiple layers along the vertical axis (i.e., \( Z \) axis). And each layer
Consists of a set of cells each of which is a hexagonal prism, as shown in Fig. 2(b). From the top-to-down view, the cells in a layer form hexagon rings. As shown in Fig. 2(c), the hexagon rings of the middle layer is centered at the source node S (represented as ◎) that initiates the broadcast of a packet. From the source node and along the vertical axis, there is a starting node nearest to the center of a cell for each upper or lower layer. Only the starting nodes, the vertex nodes nearest to hexagon centers (represented as ●) and specific vertex nodes nearest to some hexagon vertexes (represented as ○) need to retransmit the packet. We carefully analyze the transmission efficiency of our proposed protocol and other polyhedron-filling approaches using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons. As we will show, the transmission efficiency of the proposed protocol can reach 1/π. Compared with the other polyhedron-filling approaches, our proposed protocol is with the highest transmission efficiency.

The rest of this paper is organized as follows. In Section 2, we introduce some related work. In Section 3, we present the proposed protocol. In Section 4, we analyze transmission efficiency of the proposed protocol and compare the analyzed result with those of polyhedron-filling based approaches. And finally, some concluding remarks are drawn in Section 5.

Fig. 2 Illustration of partitioning the 3D space into multi-layer hexagon prisms of the hexagon ring pattern

2. Related Work

The paper [7] uses a cube as a fundament block to model a three dimensional network. It uses a cube as the approximation of a sphere of a node’s transmission space on the basis of the fact that a 3D space can be quantized by a set of cubes in geometry. The papers [2] and [3] study the base station
arrangement problem for 3D picocellular networks. They rely on the hexagonal prism pattern and the rhombic dodecahedron pattern, respectively, for positioning base stations. The paper [1] studies the sensor deployment problem for 3D wireless sensor networks. The problem is to select the minimum number of sensors for covering a given space, while keeping sensors connected. The solution can be solved on the basis of Kelvin’s conjecture. In 1887, Kelvin raised a question: “What is the optimal way to fill a three dimensional space with cells of equal volume, so that the surface area (interface area) is minimized?” Kelvin merely conjectured (but did not proved) that using truncated octahedrons to fill the space is optimal. In [1], truncated octahedrons are used for 3D sensor network deployment. Moreover, cubes, hexagonal prisms and rhombic dodecahedrons are also applied to the deployment for the sake of comparison. The structures are analyzed in terms of the number of cells needed to cover a given space. By the analysis, the truncated octahedron indeed presents the minimum number of nodes for deployment. The paper [16] proposes using the body-center cubic (BCC) structure for 3D sensor network deployment. By BCC, sensors are deployed at eight vertexes of a cube and at the center of the cube. As shown in [1], the BCC-based deployment is equivalent to truncated octahedron-based deployment by the Voronoi tessellation analysis.

The paper [4] proposes a 3D broadcast protocol, called air to air communication protocol (AACP), which partitions the space into multiple layers, each of which is further partitioned to be regular hexagon lattices, as shown in Fig. 3 (a). Only the nodes at (or nearest to) hexagon vertexes need to retransmit the broadcast packet. To take the scenario in Fig. 3 (a) for an example, a source node (S) initially issues the broadcast packet destined to locations (1), (2), and (3). The node nearest to the location (1) is responsible for retransmitting the packet. Then, the node nearest to the location (11) is responsible for the retransmission. The nodes nearest to the locations (111) and (112) are then responsible for the retransmission. Similarly, the other nodes nearest to the locations of hexagon vertexes are responsible for the retransmission and hence one layer of a plane is fully covered. As shown in Fig. 3(b), the source node S also triggers a forward node on each upper or lower layer plane to start the retransmission processes for the purpose that all nodes in the space can receive the broadcast packet. Note that the paper [14] also reports a similar protocol.
Fig. 3 Illustration of the broadcast protocol AACP proposed in [4, 14]

The AACP protocol is transmission-efficient, as it partitions the space into multiple layers and tries to minimize the number of retransmissions on a layer plane by selecting a small number of strategic nodes to retransmit the broadcast packet. The selection of strategic nodes is based on the classical two-dimensional circle covering problem in [9], which asks “How to arrange circles such that the minimum number of circles can completely cover a given area?” The paper [9] showed that the regular hexagon lattice based arrangement is the best solution to the two-dimensional circle covering problem. AACP bases on such a hexagon lattice to select the nodes located at the hexagon vertices as the strategic nodes to keep as small as possible the number of strategic nodes and to keep the nodes’ connectivity. Although AACP is very efficient in selecting the strategic nodes for rebroadcasting on a plane, there exists uncovered space between two planes, as illustrated in Fig.3(c). In the Appendix, we show the ratio of the uncovered space is about 6% by simulation experiments using Matlab with the best environment setting suggested by [4] (i.e., with the hexagon side length being the transmission range R and the distance between two layer planes being 0.8R). We can certainly reduce the distance between two layer planes to eliminate the uncovered space problem. However, the number of overall strategic nodes is thus increased, and the transmission efficiency is in turn reduced. It remains open for AACP to decide the optimal value of the distance between two layer planes such that the network space is fully covered and the transmission efficiency is maximized.

3. The Proposed Protocol

In this section, we describe the proposed protocol, 3-Dimensional Optimized Broadcast Protocol (3DOBP). We assume that each node is aware of its location but is not aware of the network topology. We also assume that all nodes are equipped with omnidirectional antennas, whose transmission ranges are of the same value R and are modeled as spheres of the radius R.

3.1. Overview of 3DOBP
In 3DOBP, the entire network space is divided into multiple layers of height $H$. Each layer consists of a set of cells, as shown in Fig. 4(a). As shown in Fig. 4(b), each cell is a hexagonal prism of the side length $L$ and the height $H$. Fig. 4(c) shows that the source node or the initial starting node (represented as ◎) is in the middle layer (layer 0) to broadcast a packet and the interlayer nodes (represented as ⊗) is between layers to forward the packet to other starting nodes (also represented as ◎) in different layers (layers 1, -1, ..., etc.) At a specific layer, only the nodes nearest to hexagon centers (represented as ●) and the nodes nearest to some specific hexagon vertexes (represented as ○) need to retransmit the packet. Note that below we use the term center node (resp., vertex node) to stand for the node nearest to a hexagon center (resp., vertex).

We assume there is a node with transmission radius $R$ located at the center of a hexagonal prism of side length $L$ and height $H$. We can draw a circumsphere of radius $Rc$ for the hexagonal prism. The relationship of $L$, $H$ and $Rc$ is $Rc = \sqrt{L^2 + H^2}/4$. We set $V$-Ratio to represent ratio of the volume of the hexagonal prism over the volume of the circumsphere as follows.

$$V\text{-Ratio} = \frac{3\sqrt{3}}{2} L^2 H / \left( \frac{4\pi}{3} \left( \sqrt{L^2 + H^2} \right)^3 \right)$$  \hspace{1cm} (1)

When $V$-Ratio is maximized, the volume of the hexagonal prism is maximized. So, we have to derive the first derivative of $V$-Ratio equation, and then find its solution. After some basic calculations, we have the solution

$$H = \sqrt{\frac{8}{3}} L$$  \hspace{1cm} (2)

We set the transmission radius $R$ to be the circumsphere radius $Rc$. We then have

$$R = Rc = L\sqrt{\frac{3}{2}}$$  \hspace{1cm} (3)

$$L = \frac{R}{\sqrt{3/2}}$$  \hspace{1cm} (4)

$$H = L\sqrt{\frac{2}{3}} = \frac{R}{\sqrt{3/2}} \sqrt{2} = 2 \frac{R}{\sqrt{3}}$$  \hspace{1cm} (5)

To sum up, we can use a hexagonal prism of side length $\frac{R}{\sqrt{3/2}}$ and height $2 \frac{R}{\sqrt{3}}$ to represent the effective transmission space of a node of transmission radius $R$.

To broadcast a packet throughout the entire network space, we need to achieve the following two goals: (a) to activate starting nodes for retransmission in every layer along the $Z$-axis, and (b) to activate center nodes in every layer for retransmission. Below, we show how to achieve the two goals.
**How to activate starting nodes in different layers**

We partition the network space into multiple layers. The source node or the initial starting node $S_0$, which initiates the broadcast of a packet, is assumed to be in the layer 0. $S_0$ is required to activate the starting node $S_t$ in every layer $t$ with the help of interlayer nodes. $S_0$ just transmits the packet to its two neighboring interlayer nodes $I_1$ and $I_{-1}$ in layer 1 and layer -1, respectively. Then, $I_1$ will forward the packet to starting node $S_1$ in layer 1 to activate it. Similarly, $I_{-1}$ will forward the packet to the starting node $S_{-1}$ in layer -1 to activate it. Likewise, stating nodes $S_1$ (or $S_{-1}$) will active starting node $S_2$ (or $S_{-2}$), and so on. So, the starting node in every layer will be activated along the Z-axis.

![Fig. 4 Illustration of space partitioning of 3DOBP](image)

![Fig. 5 Illustration of activation of starting nodes along the Z-axis and activation of center nodes in the layer 0](image)
**How to activate center nodes in a layer**

As shown in Fig. 5, each layer forms hexagonal rings that can fully cover a layer when the rings expand. Hence, if all center nodes are activated to forward the broadcast packet, then all the nodes in the layer can receive the broadcast packet. Note that not all vertex nodes need to be activated. They are activated not for the purpose of covering the layer but for the purpose of forwarding the broadcast packet to center nodes. As we will show, a center node can rely on a vertex node to deliver packets to two center nodes. Thus, the number of activated vertex nodes is just half the number of activated center nodes.

As shown in Fig. 5, the source node $S_0$ in layer 0 first activates three vertex nodes $V_{0,1,0}$, $V_{0,1,1}$, $V_{0,1,2}$ and two interlayer nodes $I_1$, $I_1$. One vertex node then activates two center nodes. For example, the vertex node $V_{0,1,0}$ activates two center nodes $C_{0,1,0}$ and $C_{0,1,1}$. Similarly, the vertex nodes $V_{0,1,1}$ and $V_{0,1,2}$ activate center nodes $(C_{0,1,2}, C_{0,1,3})$ and $(C_{0,1,4}, C_{0,1,5})$, respectively. We have mentioned how $S_0$ activates other starting nodes in different layers with the help of interlayer nodes. Below, we describe how $S_0$ activates all center nodes in a layer with the help of hexagon rings.

The hexagon rings have only one hexagon in the central (level-0) ring, and have six hexagons in the level-1 ring, and so on. In general, there are $6k$ hexagons in the level-$k$ ring. A hexagon center in layer $t$ and in the level-$k$ ring is denoted as $C_{t,k,i}$, where $i$ is an index ranging from 0 to $6k-1$. Centers indexed by 0 lie on the horizontal axis staring from $S_t$ towards right, while other centers are indexed counterclockwise (see Fig. 6).

![Illustration of hexagon rings (in layer t) of 3DOBP](image-url)
The relative location $LC_{t,k,i}$ of $C_{t,k,i}$ relative to $S_t$ can be derived handily by a geometric mapping $M(C_{t,k,i}) \rightarrow LC_{t,k,i}$. The geometric mapping will be well defined in subsection 3.2. Additionally, the LS$_t$ is $LC_{t,0,0}$ and the LI$_t$ is $(LC_{t-1,0,0} + LC_{t,0,0})/2$ for $t>0$ (on the other hand, LI$_t$ is $(LC_{t+1,0,0} + LC_{t,0,0})/2$ for $t<0$), where LS$_t$ and LI$_t$ respectively denote the relative locations of starting node $S_t$ and interlayer node $I_t$, which are relative to the absolute location of the source node $S_t$.

In 3DOBP, the source node $S_0$ (associated with $C_{t,0,0}$) should send the broadcast packet and activate six center nodes (associated with $C_{t,1,0}, \ldots, C_{t,1,5}$) in the level-1 ring to forward the packet. And each center node associated with $C_{t,k,i}$ in the level-$k$ ring, $k \geq 1$, should either activate no node or activate two neighboring center nodes in the next level. Actually, for $k \geq 1$, $3(k+1)$ center nodes in the level-$k$ ring need to activate 2 neighboring center nodes in the level-$(k+1)$ ring, while $3(k-1)$ nodes need not to activate any node. For example, all 6 level-1 center nodes need to activate 2 level-2 center nodes, and thus all 12 level-2 center nodes can be activated properly. For another example, 9 (resp., 3) out of 12 level-2 center nodes need to (resp., need not to) activate 2 level-3 center nodes, and thus all 18 level-3 center nodes can be activated properly. We devise a mapping called the activation target mapping $T(C_{t,k,i})$ that outputs an empty set or a set \{ $C_{t,k+1,w}, C_{t,k+1,w+1}$ \} of two next-level neighboring center nodes for $C_{t,k,i}$, $k \geq 1$, to activate. Note that $C_{t,k+1,w}$ (resp., $C_{t,k+1,w+1}$) must be a neighboring center node of $C_{t,k,i}$, i.e., the associated hexagons of $C_{t,k,i}$ and $C_{t,k+1,w}$ (resp., $C_{t,k+1,w+1}$) must share an edge. The activation target mapping will be well defined in subsection 3.3.

By the node activation process just mentioned, all center nodes can be activated to transmit the packet to cover the entire layer. However, since two center nodes cannot communicate with each other directly, we need intermediate nodes between them for relaying the packet. 3DOBP chooses vertex nodes (i.e., the node nearest to a hexagon vertex) as the intermediate nodes to take the advantage that a vertex node can reach two center nodes (e.g., $V_{t,1,0}$ can reach $C_{t,1,0}$ and $C_{t,1,1}$). In 3DOBP, $S$ takes 3 vertex nodes associated with $V_{t,1,0}$, $V_{t,1,1}$, and $V_{t,1,2}$ as intermediate nodes, while the other center node associated with $C_{t,k,i}$ takes only 1 (or 0) vertex node associated with $V_{k+1,i}$. The relative location $LV_{t,k,i}$ of $V_{t,k,i}$, $k>1$, relative to $S$ can be derived by computing the location of the center of $C_{t,k-1,i}$, $C_{t,k,w}$, and $C_{t,k,w+1}$ if $C_{t,k-1,i}$ should activate $C_{t,k,w}$ and $C_{t,k,w+1}$ (i.e., $T(C_{t,k-1,i})=\{ C_{t,k,w}, C_{t,k,w+1} \}$). Note that for $k=1$, $LV_{t,1,i}$ is the location of the center of $S$, $C_{t,1,2i}$, and $C_{t,1,2i+1}$ for $i=0,1,2$.

The broadcast packet of 3DOBP is of the format $P(LS, F, FA)$, where LS is the absolute location of the source node, and F is the set of relative locations of intended forwarding nodes in the next-level ring. And, FA is the set of relative locations of intended interlayer (or starting) nodes in the
neighboring layers. Note that each packet is sent along with a unique packet ID so that a node can decide if the packet has ever been received. Also note that the relative locations are delivered along with the indexes of center nodes or vertex nodes. That is, when a location $LC_{t,k,i}$ or $LV_{t,k,i}$ is delivered, the indexes $t$, $k$ and $i$ are also delivered in the packet. Those indexes are very important for a node to calculate the relative locations of intended forwarding nodes by the activation target mapping and the geometric mapping.

3D Optimized Broadcast Protocol (3DOBP)

The step for the source node $S$ to broadcast a packet $P$

1. $S$ sends the packet $P(LS, F, FA)$ with $F=\{LV_{0,1,0}, LV_{0,1,1}, LV_{0,1,2}\}$, $FA=\{LI_1, LI_{-1}\}$.

Steps for other node $X$ receiving $P(LS, F, FA)$:

1. If $X$ receives $P$ for the first time, it registers $P$. Otherwise, it drops $P$ and stops.
2. If $X$ is not a node nearest to a location in $F$ or $FA$, it stops.
3. If $X$ is nearest to a center node associated with $C_{t,k,i}$ of a location in $F$ and $T(C_{t,k,i}) \neq \emptyset$, then $X$ sends $P(LS, F')$ and stops, where $F'=\{LV_{t,k,i}\}$.
4. If $X$ is nearest to a vertex node associated with $V_{t,k,i}$ of a location in $F$, $X$ sends $P(LS, F', \emptyset)$ and stops. Indeed, $X$ can calculate $T(C_{t,k-1,i})=\{C_{t,k,w}, C_{t,k,w+1}\}$ based on indexes $k$, $i$ and then set $F'=\{LC_{t,k,w}, LC_{t,k,w+1}\}$.
5. If $X$ is nearest to an interlayer node associated with $I_t$ of a location in $FA$, $X$ sends $P(LS, \emptyset, FA')$ and stops. $X$ set $FA'=\{LS_{t+1}\}$ if $t$ is positive (i.e. up-direction activation). Otherwise, $X$ set $FA'=\{LS_{t-1}\}$ if $t$ is negative (i.e. down-direction activation).
6. If $X$ is nearest to a starting node associated with $S_t$ of a location in $FA$, $X$ sends $P(LS, F', FA')$ and stops. Indeed, $X$ can calculate and then set $F'=\{LV_{t,1,0}, LV_{t,1,1}, LV_{t,1,2}\}$. $X$ set $FA'=\{LI_{t+1}\}$ if $t$ is positive (i.e. up-direction activation). Otherwise, $X$ set $FA'=\{LI_{t-1}\}$ if $t$ is negative.

The article [13] proposes two mechanisms to determine whether a node $X$ is the one nearest to a given location. The first mechanism is to make nodes periodically exchange location information with neighboring nodes so that each node can properly elect the node nearest to the given location. The second mechanism is to enforce a backoff timer which is inversely proportional to the distance between a node’s location and the given location. The node nearest to the given location thus has the
shortest backoff timer and will earliest issue a response, which in turn prohibits other nodes from responding. When node density is sufficiently high, it is usual that only the node closest to the selected location would retransmit the packet. 3DOBP adopts the second mechanism to determine whether a node is the one nearest to a given location. In this way, a node does not need to exchange location information periodically; a node needs only its own location information for 3DOBP to run properly.

3.2. Geometric Mapping

In this subsection we present the geometric mapping \( M(C_{t,k,i}) \) that maps a hexagon center node \( C_{t,k,i} \) to a location \( L_{t,k,i} \) relative to the source node \( S \). Let \( Z_{t,0,0} \) denote the location relative to \( S \) in the \( t \)-layer. As shown in Fig. 7, each hexagon ring can be partitioned into six sectors, indexed by 0,...,5, with each sector having \( k \) hexagon centers in the level-\( k \) ring. Let \( Z_{t,k,q} \) denote the location relative to \( S \) of the first hexagon center in the sector \( q \) of the level-\( k \) hexagon ring in the layer \( t \) (e.g., \( Z_{1,2,0} \) is the location of the hexagon center on the horizontal line from \( S \) towards right in the layer 1). We have \( Z_{t,k,q} = (H \cdot t, k\sqrt{3}L \cdot \cos(q \cdot 60^\circ), k\sqrt{3}L \cdot \sin(q \cdot 60^\circ)) \) for \( q=0,...,5 \), where \( L \) is the hexagon side length determined by Eq. (4), and \( H \) is the height of hexagon prism determined by Eq. (5). Since each sector has \( k \) hexagon centers, we can figure out that hexagon center \( C_{t,k,i} \) is within sector \( q \), where \( q = \lfloor i/k \rfloor \). Now we can define the geometric mapping \( M(C_{t,k,i}) \) as follows. (Note that “+” represents the vector addition operator in the mapping and the following location calculations.)

Let \( q = \lfloor i/k \rfloor \). If \( i \) is a multiple of \( k \), \( M(C_{t,k,i}) = Z_{t,k,q} \).

Otherwise, \( M(C_{t,k,i}) = \begin{cases} 
Z_{t,k,0} + (0, -i \frac{\sqrt{3}L}{2}, i \frac{3L}{2}), & \text{if } q = 0 \\
Z_{t,k,1} + (0, (k-i)\sqrt{3}L, 0), & \text{if } q = 1 \\
Z_{t,k,2} + (0, (q \cdot k - i) \frac{\sqrt{3}L}{2}, (q \cdot k - i) \frac{3L}{2}), & \text{if } q = 2 \\
Z_{t,k,3} + (0, (i - q \cdot k) \frac{\sqrt{3}L}{2}, (i - q \cdot k) \frac{3L}{2}), & \text{if } q = 3 \\
Z_{t,k,4} + (0, (i - q \cdot k)\sqrt{3}L, 0), & \text{if } q = 4 \\
Z_{t,k,5} \left(0, (i - q \cdot k) \frac{\sqrt{3}L}{2}, (i - q \cdot k) \frac{3L}{2}\right), & \text{if } q = 5 
\end{cases} \)

In Fig. 7, we illustrate the above mapping by two examples. The first example is about \( M(C_{0,2,1}) \). Since \( q=[1/2]=0 \), we calculate \( Z_{0,2,0} = \left(0, 2\sqrt{3}L \cdot \cos(0), 2\sqrt{3}L \cdot \sin(0)\right) = (0, 2\sqrt{3}L, 0) \). We then
have $M(C_{0,2,1}) = Z_{0,2,0} + \left(0, -\frac{\sqrt{3}L}{2}, \frac{3L}{2}\right)$. The second example is about $M(C_{0,2,7})$. Since $q=[7/2]=3$, we calculate $Z_{0,2,3} = \left(0, 2\sqrt{3}L \cdot \cos(180^\circ), 2\sqrt{3}L \cdot \sin(180^\circ)\right) = (0, -2\sqrt{3}L, 0)$. We then have $M(C_{0,2,7}) = Z_{0,2,3} + \left(0, \frac{\sqrt{3}L}{2}, -\frac{3L}{2}\right)$.

### 3.3. Activation Target Mapping

In this subsection we present the activation target mapping $T(C_{t,k,i})$. The input of $T(C_{t,k,i})$ is a center node $C_{t,k,i}$ for $k \geq 1$. $T(C_{t,k,i})$ is to find two next-level neighboring center nodes $C_{t,k+1,w}$ and $C_{t,k+1,w+1}$ of $C_{t,k,i}$, where $w$ is even. If such neighboring nodes exit, the output of $T(C_{t,k,i})$ is $\{C_{t,k+1,w}, C_{t,k+1,w+1}\}$; otherwise, the output is an empty set. For example, $T(C_{0,1,0}) = \{C_{0,2,0}, C_{0,2,1}\}$ since $C_{t,k+1,w}$ is $C_{0,2,0}$ and $C_{t,k+1,w+1}$ is $C_{0,2,1}$ with $w=0$. For another example, $T(C_{0,2,1}) = \emptyset$, since the index $w$ of the neighboring center nodes ($C_{0,3,1}$ and $C_{0,3,2}$) of $C_{0,2,1}$ is odd.

As shown in Fig. 7, each hexagon ring can be partitioned into six sectors, indexed by $0,\ldots,5$, each having a starting axis (i.e., $A_0$, $\ldots$, $A_5$). Let $q=[i/k]$ denote the index of the sector in which $C_{t,k,i}$ resides. $T(C_{t,k,i})$ is defined as follows.

$$T(C_{t,k,i}) = \begin{cases} 
\{C_{t,k+1,i}, C_{t,k+1,i+1}\}, & \text{if } q = 0 \text{ and } i \text{ is even} \\
\{C_{t,k+1,i+1}, C_{t,k+1,i+2}\}, & \text{if } q = 1 \text{ and } i \text{ is odd} \\
\{C_{t,k+1,i}, C_{t,k+1,i+1}\}, & \text{if } q = 1, (i \mod k) = 0 \text{ and } i \text{ is even} \\
\{C_{t,k+1,i+2}, C_{t,k+1,i+3}\}, & \text{if } q = 2 \text{ and } i \text{ is even} \\
\{C_{t,k+1,i+3}, C_{t,k+1,i+4}\}, & \text{if } q = 3 \text{ and } i \text{ is odd} \\
\{C_{t,k+1,i+2}, C_{t,k+1,i+3}\}, & \text{if } q = 3, (i \mod k) = 0 \text{ and } i \text{ is even} \\
\{C_{t,k+1,i+4}, C_{t,k+1,i+5}\}, & \text{if } q = 4 \text{ and } i \text{ is even} \\
\{C_{t,k+1,i+5}, C_{t,k+1,i+6}\}, & \text{if } q = 5 \text{ and } i \text{ is odd} \\
\{C_{t,k+1,i+4}, C_{t,k+1,i+5}\}, & \text{if } q = 5, (i \mod k) = 0 \text{ and } i \text{ is even} \\
\emptyset, & \text{otherwise}
\end{cases}$$

The definition of $T(C_{t,k,i})$ contains 10 cases of different conditions. The first case ($q=0$ and $i$ is even) drives the center node $C_{t,k,i}$ to activate two neighboring center nodes $C_{t,k+1,i}$ and $C_{t,k+1,i+1}$ in the next level, only if $C_{t,k,i}$ is in sector 0 and $i$ is even. The second case ($q=1$ and $i$ is odd) drives $C_{t,k,i}$ to activate $C_{t,k+1,i+1}$ and $C_{t,k+1,i+2}$, only if $C_{t,k,i}$ is in sector 1 and $i$ is odd. The third case ($q=1$, $(i \mod k)=0$ and $i$ is even) drives $C_{t,k,i}$ to activate $C_{t,k+1,i}$ and $C_{t,k+1,i+1}$, only if $C_{t,k,i}$ is in sector 1, $C_{t,k,i}$ is on the starting axis of sector 1 (i.e. $A_1$), and $i$ is even. Similarly, the fourth, ..., and the ninth cases drive $C_{t,k,i}$
in the sectors 2, ..., and 5 to activate two center nodes for specific conditions. The last case drives $C_{t,k,i}$ not to activate any node, only if none of the first nine conditions is satisfied.

Fig. 7 The illustration of the Geometric Mapping and Activation Target Mapping

In Fig. 7, we illustrate $T(C_{t,k,i})$ by three examples. The first example is about $T(C_{0,2,0})$. Let $q=[0/2]=0$. Since $q$ is 0 and $i (=0)$ is even, we have $T(C_{0,2,0})=\{ C_{0,3,0}, C_{0,3,1} \}$. The second example is about $T(C_{0,2,5})$. Let $q=[5/2]=2$. Since $q$ is 2 and $i (=5)$ is not even, we have $T(C_{0,2,5})=\emptyset$, which means the node $C_{0,2,5}$ needs not to activate any node. The third example is about $T(C_{0,2,2})$. Let $q=[2/2]=1$. Since $q$ is 1, $i (=2)$ is even, and $i$ is a multiple of $k (=2)$, we have $T(C_{0,2,2})=\{ C_{0,3,2}, C_{0,3,3} \}$.

4. Performance Analysis

In this section, we first derive the theoretical upper bound of transmission efficiency for 3D wireless networks. We then analyze and compare the transmission efficiency of the proposed 3DOBP and the 3D broadcast protocols based on filling space by polyhedrons: cubes, hexagonal prisms, rhombic dodecahedrons, and truncated octahedrons.

4.1. Transmission Efficiency Upper Bound

The transmission efficiency is defined as the effective transmission region over the sum of the transmission regions. We use two spheres $A$ and $B$ of radius $R$ in Fig. 8 to derive the transmission efficiency upper bound.

According to [17], the intersected volume of two spheres is as follows:
In Eq. (6), \( R \) is the radius of sphere and \( d \) is the distance of two spheres.

To minimize the intersected volume, while keeping the two sphere center within distance \( R \) is \( d=R \). Hence, the upper bound of transmission efficiency (TE) in the 3D space is as follows:

\[
\text{TE}(R) = \frac{2V_{\text{sphere}}(R) - V_{\text{inter}}(R,R)}{2V_{\text{sphere}}(R)}
\]  

(7)

In Eq. (7), \( V_{\text{sphere}} \) is the volume of sphere with radius \( R \), which is \( (4/3)\pi R^3 \).

Letting \( R=1 \), we have the upper bound of transmission efficiency in 3D as follows:

\[
\text{TE} = 1 - \frac{(5/12)\pi}{2(4/3)\pi} = \frac{27}{32} = 0.84375
\]  

(8)

Thus, we have that the transmission efficiency upper bound is 0.84375.

![Intersected spheres](image)

Fig. 8 Intersected spheres, where center of sphere A on the left is at (0,0,0) and sphere B on the right is at (1,0,0), and the distance between two centers is one unit distance.

### 4.2. Transmission Efficiency of Cubes

We assume a cube is with the side length \( L \). The center of each cube is located by a node of transmission radius \( R \). We can form a circumsphere for a cube, where the radius of circumsphere is \( R_c \). The relationship between the side length and the radius of circumsphere is: \( L = 2R_c/\sqrt{3} \). The distance between two neighboring cube centers should be within \( R \). Thus, \( R = L = 2R_c/\sqrt{3} \).

For a set of \( N \) nodes with the cube arrangement, the transmission efficiency is:

\[
\frac{N-L^3}{N-\pi(2R_c/\sqrt{3})^3} = \frac{8Rc^3/3\sqrt{3}}{\pi8Rc^3/3\sqrt{3}/4/3} = \frac{3}{4\pi} = 0.238732
\]  

(9)
Fig. 9 Illustration of the cube arrangement, where L is the side length, R is the transmission radius, and Rc is the radius of the circumsphere of the cube cell.

4.3 Transmission Efficiency of Hexagonal Prisms

We assume a hexagonal prism is with side length L and height H, and that the center of each hexagonal prism is located by a node with transmission radius R. By Eq. (2), the hexagon prism has the maximal volume when H= $L\sqrt{2}$. We have the volume of hexagonal prism (HPV) is \( \frac{3\sqrt{3}}{2}L^2 \cdot L\sqrt{\frac{2}{3}} = \frac{3\sqrt{3}}{\sqrt{2}}L^3 \). Moreover, we set the radius Rc of the circumsphere of the hexagonal prism as follows:

\[
R_c = \sqrt{L^2 + H^2/4} = \sqrt{L^2 + L^2/2} = \sqrt{3L^2/2} = L\sqrt{3/2}
\]

(10)

And, the relationship between the transmission radius and the circumsphere radius is R=$\sqrt{2}$Rc. We have the following equation:

\[
R = \sqrt{2}R_c = \sqrt{2}L\sqrt{\frac{3}{2}} = \sqrt{3}L
\]

(11)

For a set of N nodes with hexagonal prism arrangement, the transmission efficiency is as follows:

\[
\frac{N \cdot \text{HPV}}{N \cdot \pi R^3 \cdot 4/3} = \frac{\frac{3\sqrt{3}}{\sqrt{2}}L^3}{\pi \cdot (\sqrt{3}L)^3 \cdot 4/3} = \frac{3}{\pi \cdot (\sqrt{3})^3 \cdot 4/3} = \frac{3}{4\sqrt{2}\pi} = 0.168809
\]

(12)

Fig. 10 Illustration of the hexagonal prism arrangement

4.4 Transmission Efficiency of Rhombic Dodecahedrons

A rhombic dodecahedron can be constructed by two cubes of the length L. Based on the results of [1], we have the following settings. We can construct a circumsphere for a rhombic dodecahedron. The radius Rc of the circumsphere is L. The volume RDV of a rhombic dodecahedron is \( 2L^3 \). The transmission radius R of a rhombic dodecahedron center node is $\sqrt{2}$Rc. Since Rc=L, we have R=$\sqrt{2}$L.

For a set of N nodes with rhombic dodecahedron arrangement, the transmission efficiency is:

\[
\frac{N \cdot \text{RDV}}{N \cdot \pi R^3 \cdot 4/3} = \frac{2L^3}{\pi \cdot (\sqrt{2}L)^3 \cdot 4/3} = \frac{1}{\pi \cdot (\sqrt{2})^3 \cdot 4/3} = \frac{3}{4\sqrt{2}\pi} = 0.168809
\]

(13)
Fig. 11 Illustration of the rhombic dodecahedral cell

4.5 Transmission Efficiency of Truncated Octahedrons

We assume that a truncated octahedron is of side length \( L \) and the center of each truncated octahedron is located by a node whose transmission radius is \( R \). Let \( R_c \) be the radius of the circumsphere of a truncated octahedron. Based on the results of [1], we set \( R_c = L\sqrt{\frac{10}{2}} \) and \( R = 4R_c/\sqrt{5} \). Then we have \( R = \frac{4L\sqrt{10}}{\sqrt{5}} = 2L\sqrt{2} \).

For a set of \( N \) nodes with truncated octahedron arrangement, the transmission efficiency is:

\[
\frac{N \cdot 8\sqrt{2}L^3}{N^2 \pi (2L\sqrt{2})^3 - 4/3} = \frac{N \cdot 8\sqrt{2}}{N \pi (2\sqrt{2})^3 - 4/3} = \frac{3}{8\pi} = 0.119366
\]  

(14)

Fig. 12 Illustration of the truncated octahedron cell

4.6 Transmission Efficiency of 3DOBP

We assume a hexagonal prism is with side length \( L \) and height \( H \), and that the center of each hexagonal prism is located by a node with transmission radius \( R \). By Eqs. (4) and (5), we have \( L = \frac{R}{\sqrt{3/2}} \) and \( H = 2 \frac{R}{\sqrt{3}} \). The volume \( \text{HPV} \) of a hexagonal prism is:

\[
\text{HPV} = \frac{3\sqrt{3}}{2} L^2 \cdot H = \frac{3\sqrt{3}}{2} L^2 \cdot L\sqrt{2} = \frac{3\sqrt{3}}{\sqrt{2}} L^3
\]  

(15)

With the help of hexagon rings, in 3DOBP, all center nodes suffice to cover a layer of the space. We assume the number of center nodes is \( N \). Then we need a number of \( N/2 \) of vertex nodes to be activated to forward packets to the \( N \) center nodes. Let \( M \) be the number of center nodes and activated vertex nodes. We have

\[
N = \sum_{k=1}^{H} 6k = 3H(H + 1)
\]  

(16)
\[ M = \sum_{k=1}^{H} 6k + \frac{1}{2} \sum_{k=1}^{H} 6k = \frac{3}{2} 3H(H + 1) \] (17)

To simplify the calculation, we assume the number of interlayer nodes is ignorable. The transmission efficiency of the proposed protocol is

\[ TR = \frac{N \cdot \text{HPV}}{M \cdot \pi \cdot R^3 / 3} = \frac{3H(H+1) \cdot \text{HPV}}{2 \cdot \pi \cdot R^3 / 3} = \frac{3 \sqrt[3]{L^3}}{2 \pi (L / \sqrt[3]{2})^3} = \frac{1}{\pi} = 0.31831 \] (18)

4.7 Transmission Efficiency Comparison

In this subsection, we compare 3DOBP with other 3D broadcast protocols based on filling space by polyhedrons like cubes [7], hexagonal prisms [1, 3], rhombic dodecahedrons [1, 2], and truncated octahedrons [1] in terms of transmission efficiency. Note that the comparisons do not include the 3D broadcast protocol AACP proposed in [4], since the network space is not fully covered even when node density is sufficiently high. The comparison results are summarized in the Table 1. We can observe that 3DOBP is with the highest transmission efficiency.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Transmission efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube based</td>
<td>( \frac{3}{4\pi} \approx 0.238732 )</td>
</tr>
<tr>
<td>Hexagonal Prism based</td>
<td>( \frac{3}{4\sqrt{2}\pi} \approx 0.168809 )</td>
</tr>
<tr>
<td>Rhombic Dodecahedron based</td>
<td>( \frac{3}{4\sqrt{2}\pi} \approx 0.168809 )</td>
</tr>
<tr>
<td>Truncated Octahedron based</td>
<td>( \frac{3}{8\pi} \approx 0.119366 )</td>
</tr>
<tr>
<td>3DOBP</td>
<td>( \frac{1}{\pi} \approx 0.31831 )</td>
</tr>
</tbody>
</table>

5. Conclusion

Wireless technologies are promising for industrial automation and distributed control due to the properties of mobility, reduced deployment and maintenance costs, and so on. By the technologies, wireless networks are formed for integrating processes and devices of industrial systems. Broadcasting is a fundamental operation in the wireless network to disseminate data and/or control signals. In this paper, we have generalized the definition of transmission efficiency for 3D wireless networks and shown that the theoretical upper bound of transmission efficiency of 3D broadcast protocols is 0.84. We have further proposed an optimized broadcast protocol, called 3D optimized broadcast protocol (3DOBP), by partitioning the 3D space into multi-layer hexagonal prisms of a
hexagon ring pattern in each layer. As we have shown, the transmission efficiency of the proposed protocol can reach $1/\pi$, which is better than those of other 3D broadcast protocols based on polyhedron-filling schemes using cubes, hexagon prisms, rhombic dodecahedrons, and truncated octahedrons. We conclude that 3DOBP is very efficient for broadcasting in wireless networks for industrial systems of 3D operation environments such as multi-storey factories.

Appendix

In this appendix, AACP [4, 14] is investigated to see whether it can fully cover the entire network space. We conducted simulation experiments with the Matlab tool under the assumption that 12 rebroadcasting nodes are at the ideal strategic locations suggested by paper [4] (see Fig. 15). To be more precise, rebroadcasting nodes are deployed on two layer planes with 6 nodes begin arranged on a layer plane as a hexagonal pattern. The distance between the two layer planes is $0.8R$, where $R$ is the transmission radius of the rebroadcasting nodes. We also uniformly deploy nodes with a high density (i.e., 500 nodes per $R \times R \times R$ space) and monitor only the nodes inside the space bounded by the 12 rebroadcasting nodes. The simulation result shows that 6% of the monitored nodes do not receive the broadcast packet, which implies the coverage ratio is only about 94%. The 12 rebroadcasting nodes and the nodes not receiving the broadcast packet are depicted in Fig. 16.

Fig. 15. The deployment of 12 rebroadcasting nodes on two layer planes for AACP, where each sphere represents the covering space of a rebroadcasting node
Fig. 16 Illustration of unreachable nodes in AACP, where dot symbols represent unreachable nodes and star symbols represent rebroadcasting nodes.
References


